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Article

Quantum Game of Dual-channel Supply Chain under Free-riding Behavior

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Abstract: Based on the perspective of the quantum game, this paper explores when the online direct sales channel takes the free-riding behavior after the retail channel provides high-quality experience and services and how the dual-channel supply chain establishes a commodity pricing strategy. The retailer's selling price follows a decreasing function of the free-riding behavior coefficient. while the online direct selling price does an increasing function of the free-riding behavior coefficient. Under centralized decision-making, there is no quantum entanglement, so the quantum game solution is consistent with the classical game solution. Under decentralized decision-making, the optimal price and profit of the quantum game are higher than those of the classical game when the quantum entanglement degree is greater than zero. When the quantum entanglement tends to be infinite, the optimal price of the quantum game finally remains in convergence. The quantum game theory is a more optimal decision-making method than the classical game theory.

Keywords: Quantum Game, Dual-channel Supply Chain, Pricing Strategy, Free-riding Behavior

1. Introduction

With the advancement of science and technology, the world has entered an era of informatization such that e-commerce is booming. The vigorous development of e-commerce has promoted the popularization of online consumption. Traditional retail methods no longer meet the shopping needs of consumers. Manufacturers have opened up online direct sale channels one after another, such that the dual-channel model where retail channels and online direct sales channels coexist is developing rapidly. In the shopping environment of a dual-channel supply chain, consumers are faced with more choices of products and services than before and easily switch between different channels to meet their purchase needs (Tsay & Agrawal, 2004).

Canton and Chevalier (2001) believe that the efforts of retailers promote the occurrence of free-riding behaviors. The advertisement cost of the traditional branded products, space cost of commodity displaying, and service cost of commodity selling in offline stores are borne by manufacturers and retailers. Customers obtain the information about the product and enjoy the service, but at this time they tend to buy the product online at lower prices. This behavior of consumers weakens the enthusiasm of retailers for sales efforts and affects the manufacturers' decision-making. Wu et al. (2004) believed that information service has the characteristics of public goods, and consumers use the information service to make an informed purchase decision. After receiving the information service from an information service provider, consumers easily free ride by purchasing at low-price sellers who do not provide any information service.

There are many achievements in the research of dual-channel supply chain, and the problem of pricing decision has always attracted much research interest (Chiang et al., 2003). The methods of studying the pricing decision and coordination strategy of dual-channel supply chain mostly adopt Cournot, Stackelberg. or Bertrand classic game theory. Yao and Liu (2005) studied the Bertrand and Stackelberg price competition models in dual channels. They obtained both the Bertrand and Stackelberg equilibrium pricing policies and compared the profit gains under the two competitions. Kurata et al. (2007) explored cross-brand and cross-channel pricing policies and assed supply chain coordination. They found that the wholesale price change does not coordinate the supply chain, and an appropriate combination of markup and markdown prices achieves both supply chain coordination and a win-win outcome for each channel. Basak and Wang (2019) studied the endogenous choice of price and quantity competition in a mixed duopoly. They found that the profit of the Cournot model is significantly higher than that of the Bertrand model in the standard oligopoly and the opposite is found in the mixed oligopoly.

In 1999, Meyer proposed the theory of quantum games and found that a player using quantum strategies always defeats an opponent who uses classical strategies (Meyer, 1999). Eisert et al. (1999) introduced quantum strategy into the Prisoners' Dilemma and used quantum entanglement to eliminate the dilemma. They found that this game ceases to pose a dilemma if quantum strategies are allowed. Many studies found that quantum games solve some of the difficult problems encountered in classical game theory. There is a relationship of competition and cooperation between manufacturer and retailer in the dual-channel supply chain. The relationship of competition and cooperation creates the phenomenon of quantum entanglement. With the closer competition or cooperation between manufacturer and retailer, the degree of quantum entanglement increases. How to price a commodity to obtain the optimal profit is a more complicated problem than the classic game pricing problem, as every quantum entanglement degree corresponds to a classical game pricing problem.

This paper discusses the following problem from the perspective of a quantum game: how does the free-riding behavior of online direct sales channels affect commodity pricing, demand, and supply chain profits, and when the retail channels in the dual-channel supply chain provide services.

2. Dual-channel Supply Chain

2.1. Problem Description

The supply chain structure that consists of one manufacturer and one retailer is considered (Fig. 1). Manufacturer M has a dual-channel supply chain and sells a product through the retail channel and the online channel at the same time. In Figure 1, the product cost is c, the wholesale price of the retailer T to purchase is ω, the retail channel sales price is p_0, and the online direct sales price is p_1.

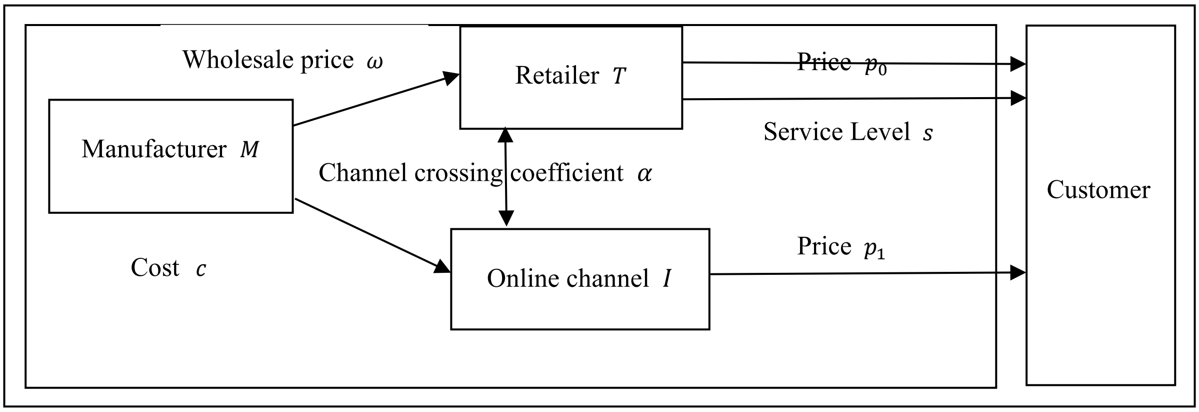

Fig. 1. The structure of the dual-channel supply chain.

2.2. Basic Hypotheses

To make the model rigorous, the following hypotheses are proposed.

Hypothesis 1. Both manufacturer and retailer are rational managers. To ensure the basic profitability of the enterprises, $0 < c < \omega < p_0, p_1$ is assumed.

Hypothesis 2. α is the channel cross-elasticity coefficient in the dual-channel supply chain, which states the degree of sensitivity of the product demand of one channel in the dual-channel supply chain to the price changes of another channel. Thus, $0 < \alpha < 1$.

Hypothesis 3. Retail channels provide more types of services than online channels such as on-site explanations, product experience, and so on. Only retail channels provide services, while online channels do not. Retailer T provides the level of service s for the product spend service cost $c(s) = \frac{1}{2}\eta s^2$, where $\eta > 0$ is the unit service cost spent by the retailer on the product.

2.3. Demand Function and Profit Function

This paper refers to the linear demand functions based on the price and service sensitivity that was established by Banker *et al.* (1988) and Huang and Swaminathan (2009) to establish the demand function. The demand functions of retailer T and manufacturer M are as follows.

$$Q_0 = a_0 - b_0 p_0 + \alpha p_1 + (1 - \lambda)s, \quad (1)$$

$$Q_1 = a_1 - b_1 p_1 + \alpha p_0 + \lambda s, \quad (2)$$

where Q_0 and Q_1 represent the demand functions for the retail and the online channels of the dual-channel supply chain and b_0 and b_1 represent the direct price elasticity coefficient for the retail and the online channel of the dual-channel supply chain. When a_j is positive and stable, representing the potential market size corresponding to Q_j, $j = 0, 1$. When λ represents the free-riding behavior coefficient of online channels of the dual-channel supply chain, the parameters satisfy $b_j > 1$, $j = 0, 1$, and $0 < \lambda < 1$.

According to the problem description, basic hypotheses and demand functions, the total profit function of the retailer T, the total profit functions of manufacturer M are represented by π_0 and π_1, respectively as follows.

$$\pi_0 = (p_0 - \omega)\, Q_0 - \frac{1}{2}\eta\, s^2, \tag{3}$$

$$\pi_1 = (\omega - c)\, Q_0 + (p_1 - c)\, Q_1, \tag{4}$$

The overall supply chain profit π_{SC} of retailer T and manufacturer M is

$$\pi_{SC} = (p_1 - c)Q_1 + (p_0 - c)Q_0 - \frac{1}{2}\eta\, s^2. \tag{5}$$

2.4. Solutions of the Classic Game

Using classical game theory to solve the optimal profit of each player and the overall supply chain, the following relevant decision-making strategies is obtained.

2.4.1. Decentralized Decision-making

According to the first derivative condition of the profit functions $\frac{\partial \pi_j}{\partial p_j} = 0$, $j = 0, 1$, we obtain $AP = D$, where

$$A = \begin{bmatrix} 2b_0 & -\alpha \\ -\alpha & 2b_1 \end{bmatrix},\ P = \begin{bmatrix} p_0 \\ p_1 \end{bmatrix},\ D = \begin{bmatrix} a_0 + (1-\lambda)s + b_0\omega \\ a_1 + \lambda s + \alpha\omega - \alpha c + b_1 c \end{bmatrix}.$$

According to the second derivative condition of the profit functions $\frac{\partial^2 \pi_0}{\partial p_0^2} = -2b_0 < 0$ and $\frac{\partial^2 \pi_1}{\partial p_1^2} = -2b_1 < 0$, this classic game model allows the optimal prices.

The determinant of matrix A is $|A| = 4b_0 b_1 - \alpha^2 > 0$, as $b_j > 1$, $j = 0, 1$, and $0 < \alpha < 1$. We, then, obtain the following optimal prices by the Cramer's rule.

$$\bar{p}_0^* = \frac{2b_1[a_0 + (1-\lambda)s + b_0\omega] + (a_1 + \lambda s + b_1 c)\alpha + (\omega - c)\alpha^2}{4b_0 b_1 - \alpha^2}, \tag{6}$$

$$\bar{p}_1^* = \frac{2b_0(a_1 + \lambda s + b_1 c) + \alpha[a_0 + (1-\lambda)s + b_0(3\omega - 2c)]}{4b_0 b_1 - \alpha^2}. \tag{7}$$

Substituting the optimal prices into Equations (3) and (4), we obtain the optimal profit of each game player.

2.4.2. Centralized Decision-making

According to the first derivative condition of the overall profit of the supply chain $\frac{\partial \pi_{sc}}{\partial p_j} = 0$, $j = 0, 1$, and the Hessian matrix $H = \begin{bmatrix} -2b_0 & 2\alpha \\ 2\alpha & -2b_1 \end{bmatrix}$, the optimal price is obtained as follows.

$$\bar{p}_0^{c*} = \frac{1}{|A'|}\begin{vmatrix} a_0 + (1-\lambda)s + (b_0 - \alpha)c & -2\alpha \\ a_1 + \lambda s + (b_1 - \alpha)c & 2b_1 \end{vmatrix}, \tag{8}$$

$$\bar{p}_1^{c*} = \frac{1}{|A'|}\begin{vmatrix} 2b_0 & a_0 + (1-\lambda)s + (b_0 - \alpha)c \\ -2\alpha & a_1 + \lambda s + (b_1 - \alpha)c \end{vmatrix}, \tag{9}$$

where $A' = \begin{bmatrix} 2b_0 & -2\alpha \\ -2\alpha & 2b_1 \end{bmatrix}$, $|A'| = 4b_0b_1 - 4\alpha^2$, i.e.,

$$\bar{p}_0^{C*} = \frac{b_1[a_0+(1-\lambda)s+b_0c]+\alpha(a_1+\lambda s-c\alpha)}{2(b_0b_1-\alpha^2)}, \quad (10)$$

$$\bar{p}_1^{C*} = \frac{b_0(a_1+\lambda s+b_1c)+\alpha[a_0+(1-\lambda)s-c\alpha]}{2(b_0b_1-\alpha^2)}. \quad (11)$$

Substituting the optimal prices into Equation (5), we obtain the optimal overall profit of the supply chain.

3. Quantum Game Model

A quantum game model in Fig. 2 is considered in this study.

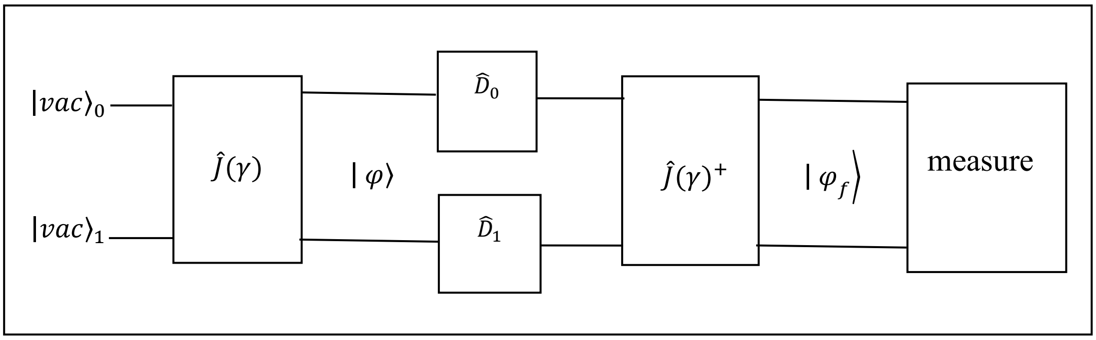

Fig. 2. Quantum structure of dual-channel supply chain.

When a manufacturer and a retailer start the game from the quantum state $|vac\rangle_0 \otimes |vac\rangle_1$. the initial state is transformed into the following quantum entangled state

$$|\varphi\rangle = \hat{J}(\gamma)(|vac\rangle_0 \otimes |vac\rangle_1), \quad (12)$$

through a specific unitary operator

$$\hat{J}(\gamma) = exp\{\, i\,\gamma\big(\hat{X}_0\hat{P}_1 + \hat{X}_1\hat{P}_0\big)\,\}, \quad (13)$$

where $\hat{X}_j = \frac{1}{\sqrt{2}}(\hat{a}_j^+ + \hat{a}_j)$, $\hat{P}_j = \frac{i}{\sqrt{2}}(\hat{a}_j^+ - \hat{a}_j)$, $j = 0, 1$, and $i = \sqrt{-1}$.

After the player selects the strategy, the strategy operator is represented by the positive operator *D* and *E*, *x* and *y* represent the strategy of the game, *Dx*=*y*.

The quantum entanglement state is determined when the player chooses different strategies from the strategy set S_j $(j = 0, 1)$. The strategy operator is represented by the unitary operators $\hat{D}_0$ and $\hat{D}_1$, where

$$\hat{D}_j(x_j) = exp\ \big(-ix_j\hat{P}_j\big), \quad (14)$$

$$S_j = \big\{\,\hat{D}_j(x_j) = exp\ \big(-ix_j\hat{P}_j\big)\,\big|\, x_j \in (-\infty, +\infty)\big\}. \quad (15)$$

By the action of the operator $\hat{J}(\gamma)^+$, when the game is over, the final state is *J*=1. After the game is over, the final state is expressed as

$$|\varphi_f\rangle = \hat{J}(\gamma)^+(\hat{D}_0 \otimes \hat{D}_1) \cdot \hat{J}(\gamma)(|vac\rangle_0 \otimes |vac\rangle_1). \quad (16)$$

By the measurement device, the relationship between the player's quantum strategy and the price is defined as

$$p_0(x_0, x_{1,}) = x_0 \cosh\gamma + x_1 \sinh\gamma, \tag{17}$$

$$p_1(x_0, x_{1,}) = x_1 \cosh\gamma + x_0 \sinh\gamma, \tag{18}$$

where γ is the degree of quantum entanglement, x_0 and x_1 represent the strategies in the quantum game, and $\sinh\gamma = \frac{e^{\gamma}-e^{-\gamma}}{2}$, $\cosh\gamma = \frac{e^{\gamma}+e^{-\gamma}}{2}$.

3.1. Decentralized Decision-making in Quantum Game

Substituting Equations (17) and (18) of the relationship between quantum strategies and prices into Equations (1) and (2) leads to the demand functions of retailer and manufacturer as follows.

$$Q_0 = x_0(-b_0 \cosh\gamma + \alpha \sinh\gamma) + x_1(-b_0 \sinh\gamma + \alpha \cosh\gamma) + a_0 + (1-\lambda)s, \tag{19}$$

$$Q_1 = x_0(-b_1 \sinh\gamma + \alpha \cosh\gamma) + x_1(-b_0 \cosh\gamma + \alpha \sinh\gamma) + a_1 + \lambda s. \tag{20}$$

Under decentralized decision-making, when manufacturer and retailer have an equal status in price decision-making, each game player aims to maximize the profit for the quantum game, then the optimal prices solutions p_0^* and p_1^* are obtained as the following theorem.

Theorem 1. Under the influence of the free-riding behavior of online direct sales channels, the optimal prices of the quantum game between retailer T and manufacturer M in decentralized decision-making are calcucated as follows.

$$p_0^* = \frac{1}{|B|}\{[2b_1(a_0 + (1-\lambda)s + b_0\omega) + (a_1 + \lambda s + b_1 c)\alpha + (\omega - c)\alpha^2]\cosh^2\gamma + \omega\alpha^2 \sinh^2\gamma$$
$$+[(b_0 c - 2b_0\omega - 2b_1\omega - a_0 - (1-\lambda)s)\alpha - c\alpha^2]\cosh\gamma \sinh\gamma\}, \tag{21}$$

$$p_1^* = \frac{1}{|B|}\{[2b_0(a_1 + \lambda s + b_1 c) + (a_0 + (1-\lambda)s - 2b_0 c + 3b_0\omega)\alpha]\cosh^2\gamma + (b_0(\omega - c)\alpha + c\alpha^2)\sinh^2\gamma + [$$
$$-2b_0^2(\omega - c) - (a_1 + \lambda s + 2b_0 c + b_1 c)\alpha$$
$$-(2\omega - c)\alpha^2)]\cosh\gamma \sinh\gamma\}. \tag{22}$$

Here,

$$|B| = 4b_0 b_1 \cosh^2\gamma - 2\alpha(b_0 + b_1)\sinh\gamma\cosh\gamma - \alpha^2 \tag{23}$$

is the determinant of matrix $B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$, where

$$B_{11} = 2b_0 \cosh^2\gamma - 2\alpha \sinh\gamma\cosh\gamma,$$

$$B_{12} = 2b_0 \sinh\gamma\cosh\gamma - \alpha(\cosh^2\gamma + \sinh^2\gamma),$$

$$B_{21} = 2b_1 \sinh\gamma\cosh\gamma - \alpha(\cosh^2\gamma + \sinh^2\gamma),$$

$$B_{22} = 2b_1 \cosh^2\gamma - 2\alpha \sinh\gamma\cosh\gamma.$$

Proof. Refer to Appendix 1.

Note. (1) When $\gamma = 0$, that is, when there is no quantum entanglement state between the retailer and the manufacturer, $|B| = 4b_0b_1 - \alpha^2$, and

$$p_0^* = \frac{2b_1[a_0 + (1-\lambda)s + b_0\omega] + (a_1 + \lambda s + b_1 c)\alpha + (\omega - c)\alpha^2}{4b_0b_1 - \alpha^2} = \bar{p}_0^*,$$

$$p_1^* = \frac{2b_0(a_1 + \lambda s + b_1 c) + \alpha[a_0 + (1 - \lambda s) + b_0(3\omega - 2c)]}{4b_0b_1 - \alpha^2} = \bar{p}_1^*.$$

At this time, the solutions are consistent with Equations (6) and (7). That is, the quantum game model changes to the classical game model when the quantum entanglement $\gamma = 0$.

(2) When $\gamma \to \infty$, the optimal prices of the quantum game are calculated as

$$p_0^* = \frac{2b_1(a_0+(1-\lambda)s+b_0\omega)+[(a_1-a_0)-(1-2\lambda)s-(b_0+b_1)(2\omega-c)]\alpha+2(\omega-c)\alpha^2}{4b_0b_1-2\alpha(b_0+b_1)}, \quad (24)$$

$$p_1^* = \frac{2b_0(a_1+\lambda s+b_1c-b_0\omega+b_0c)+[(a_0-a_1)+(1-2\lambda)s+4b_0\omega-5b_0c-b_1c)]\alpha-2(\omega-c)\alpha^2}{4b_0b_1-2\alpha(b_0+b_1)}. \quad (25)$$

as the quantum entanglement between the retailer and the manufacturer increases to infinity.

Corollary 1. The free-riding behavior of the online direct sales channel affects the prices of the two channels of the dual-channel supply chain. In fact,

(1) $\frac{\partial p_0}{\partial \lambda} < 0$. Retailer's selling price p_0 is expressed as a decreasing function of λ. As the coefficient of free-riding behavior increases, the price of the retail channel decreases.

(2) $\frac{\partial p_1}{\partial \lambda} > 0$. The online direct selling price p_1 is expressed as an increasing function of λ. As the coefficient of free-riding behavior increases, the price of online direct sales channels increases.

Proof. When the derivative of λ is input to Equation (19) and (20), since $cosh^2\gamma - sinh^2\gamma = 1$, $b_0, b_1 > 1 > \alpha$, we have

$$\frac{\partial p_0}{\partial \lambda} = \frac{1}{|B|}\{(-2b_1 + \alpha)s\, cosh^2\gamma + \alpha s\, cosh\,\gamma\, sinh\,\gamma\} = \frac{1}{|B|}[(-b_1 + \alpha)se^{\gamma} - b_1e^{-\gamma}] < 0, \quad (26)$$

$$\frac{\partial p_1}{\partial \lambda} = \frac{1}{|B|}\{(2b_0 - \alpha)s\, cosh^2\gamma - \alpha s\, cosh\,\gamma\, sinh\,\gamma\} = \frac{1}{|B|}[(b_0 - \alpha)se^{\gamma} + b_0e^{-\gamma}] > 0. \quad (27)$$

Significance of management decisions: (1) With the increase in the coefficient of free-riding behavior, the free-riding behavior of online direct sales channels saves the cost of advertising and services so that the price of goods is moderately reduced. The price of online direct sales channels is reduced, which increases sales. The retailer chooses to sell goods at lower prices for competition. (2) With the increase in the coefficient of free-riding behavior, the online direct sales channel indirectly attracts customers by a large number of high-quality experiences and services provided by retailers, leading to a large increase in demand. The increase in demand also leads to an increase in price.

3.2. Centralized Decision-making in Quantum Game

Under centralized decision-making, manufacturer and retailer are regarded as a whole. They take the overall supply chain profit maximization as the goal to conduct quantum games, then we obtain the optimal price solutions p_0^{c*} and p_1^{c*} according to the following theorem.

Theorem 2. Under the influence of the free-riding behavior of online direct sales channels, the optimal prices of the quantum game between retailer T and manufacturer M in centralized decision-making are as follows.

$$p_0^{c*} = \frac{b_1(a_0+(1-\lambda)s+b_0c)+\alpha(a_1+\lambda s-c\alpha)}{2(b_0b_1-\alpha^2)} = \bar{p}_0^{c*}, \tag{26}$$

$$p_1^{c*} = \frac{b_0(a_1+\lambda s+b_1c)+\alpha(a_0+(1-\lambda)s-c\alpha)}{2(b_0b_1-\alpha^2)} = \bar{p}_1^{c*}. \tag{27}$$

Proof. Refer to Appendix 2.

Note. Under centralized decision-making, manufacturer and retailer are regarded as a whole. With the degree of quantum entanglement $\gamma = 0$, we have $sinh\,\gamma = \frac{e^{\gamma}-e^{-\gamma}}{2} = 0$, and $cosh\,\gamma = \frac{e^{\gamma}+e^{-\gamma}}{2} = 1$. The prices of centralized decision-making of the quantum game model are consistent with the prices calculated by Eequations (10) and (11) based on the centralized decision-making of the classic game model.

4. Numerical Analysis

To further understand the influence of the parameters of the decentralized and centralized decision-making models under the quantum game point of view on optimal pricing and profits, numerical simulations are carried out by using Mathematica. The parameters of the quantum game model are shown in Table 1. After calculation, the optimal prices and profits in decentralized and centralized decision-making are obtained as shown in Table 2. Under decentralized decision-making, the comparison of optimal prices and optimal profits between the quantum game and the classical game is shown in Table 3.

Table 1. The parameter values of the quantum game model.

Symbols and description	Value	Symbols and description	Value
Potential market size of retail channel a_0	200	Potential market size of online channel a_1	150
Product cost c	20	Wholesale price ω	25
Price elasticity of product in retail channel b_0	1.5	Price elasticity of product in online channel b_1	1.4
Unit service cost η	2	Service level s	1.6
Channel cross-elasticity coefficient α	0.4	Degree of quantum entanglement γ	1
The free-riding behavior coefficient λ	0.5		

By comparing the optimal prices of decentralized decision-making and centralized decision-making as in Table 2, the following results are found.

(1) The optimal price of each game player under decentralized decision-making is lower than the optimal price under centralized decision-making.

(2) Whether under decentralized decision-making or centralized decision-making, the optimal price of retailers is higher than that of online direct selling optimal price of manufacturers.

Table 2. The optimal prices and optimal profits of decentralized and centralized decision-making in the quantum game model.

Decentralized Decision Making		Centralized Decision Making	
Symbols and Description	**Value**	**Symbols and Description**	**Value**
Optimal pricing in retail channel p_0^*	89.83	Optimal pricing in retail channel p_0^{c*}	98.09
Optimal pricing for online channel p_1^*	77.48	Optimal pricing for online channel p_1^{c*}	89.10
Optimal profit in retail channel π_0^*	6,301.21	Optimal profit in retail channel π_0^{c*}	6,538.6
Optimal profit in online channel π_1^*	4,996.12	Optimal profit in online channel π_1^{c*}	4,973.41
The overall optimal profit of the supply chain π_{sc}^*	1,129,703	The overall optimal profit of the supply chain π_{sc}^{c*}	11,512

Table 3. Comparison between the quantum game and classical game.

Quantum Game (choose $\gamma = 1$) Symbols and Description	Value	Classical Game (choose $\gamma = 0$) Symbols and Description	Value
Optimal pricing in retail channel p_0^*	99.23	Optimal pricing in retail channel $\bar{p}_0^*$	89.83
Optimal pricing for online channel p_1^*	85.07	Optimal pricing for online channel $\bar{p}_1^*$	77.48
Optimal profit in retail channel π_0^*	6,394.05	Optimal profit in retail channel $\bar{\pi}_0^*$	6,301.21
Optimal profit in online channel π_1^*	5,089.61	Optimal profit in online channel $\bar{\pi}_1^*$	4,996.12
The overall optimal profit of the supply chain π_{sc}^*	11,483.7	The overall optimal profit of the supply chain $\bar{\pi}_{sc}^*$	1,129,703

By comparing the optimal prices and optimal profits of the quantum game and classical game as shown in Table 3, it is found that the optimal prices and optimal profits of the quantum game are higher than those of the classical game.

4.1. Parameter Sensitivity Analysis of Decentralized Decision-making

Exploring the influence of the changes of various parameters of decentralized decision-making on the optimal price and profit is conducive to decision-makers to formulate improvement strategies and form decisions. Under decentralized decision-making, the impact of the free-rider coefficient on optimal prices and profits is shown in Figures 3 and 4. The influence of the channel cross coefficient on optimal price and profit is shown in Figures 5 and 6.

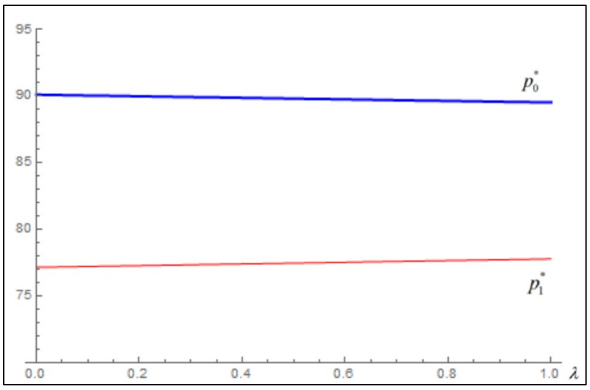

Fig. 3. The relationship between free-riding behavior coefficient and prices under decentralized decision-making.

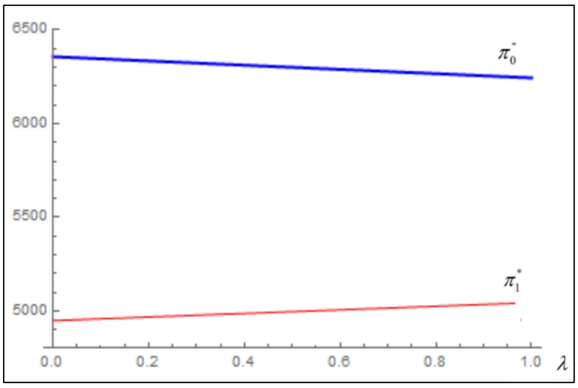

Fig. 4. The relationship between free-riding behavior coefficient and profits under decentralized decision-making.

Figs. 3 and 4 show that the price and profit functions of the online direct sales channel are increasing as they are expressed as functions of the free-riding behavior coefficient, and the price and profit functions of the retail channel are decreasing. When the free-riding behavior coefficient increases, the increasing demand for the online direct sales channel leads to an increase in prices and profits. At the same time, the reducing demand for the retail channel allows the retailer to sell at lower prices to compete and reduce inventory, which contributes to lowering the profits.

Figs. 5 and 6 present that the channel cross-elasticity coefficient has a positive impact on the optimal price and profit of each game player, which means that the price of a product of a certain channel becomes sensitive to the price of another channel in the dual-channel supply chain, and then, the demand is affected by product price fluctuations.

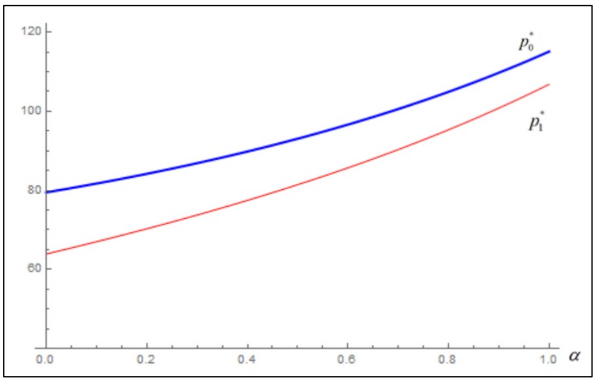

Fig. 5. The relationship between channel cross-elasticity coefficient and prices under decentralized decision-making.

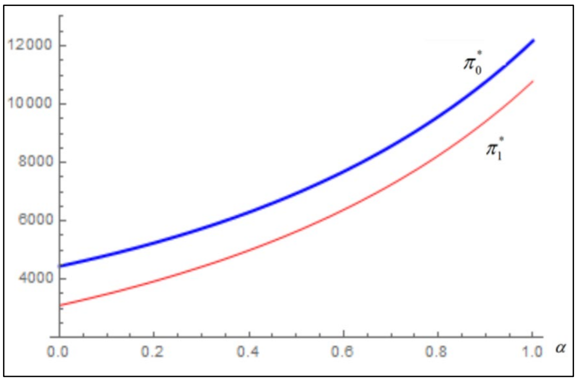

Fig. 6. The relationship between channel cross-elasticity coefficient and profits under decentralized decision-making.

4.2. Parameter Sensitivity Analysis of Centralized Decision-Making

In centralized decision-making, manufacturers and retailers are regarded altogether to maximize the overall profit of the supply chain. At this time, the free-riding behavior in the online channel provides the retail channel subsidies to strengthen the overall cooperation. Otherwise, it causes the collapse of cooperation. Then, the price of the product of the two channels falls as shown in Figure 7. The free-riding behavior in the online channel damages the profit of the retail channel to increase its profit as shown in Figure 8. Therefore, the free-riding behavior seriously harms the overall profit of the supply chain as shown in Figure 9.

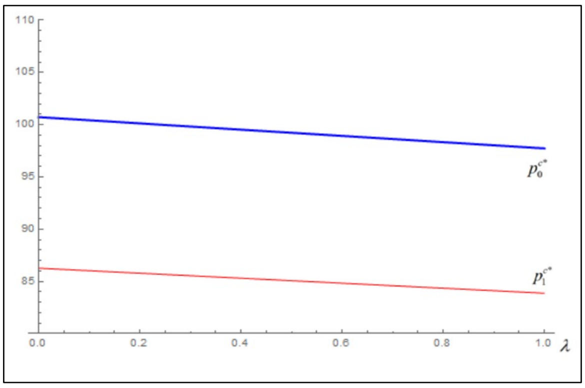

Fig. 7. The relationship between free-riding behavior coefficient and prices under centralized decision-making.

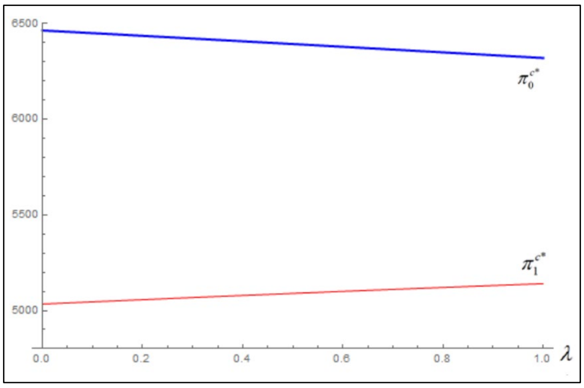

Fig. 8. The relationship between free-riding behavior coefficient and profits under centralized decision-making.

4.3. Comparison between Classical Game and Quantum Game

Under decentralized decision-making, the comparison of the optimal total profit between the quantum and the classical game in the overall supply chain is shown in Figure 10. The 3D comparison shows that the optimal profit of the overall supply chain of the quantum game is better than that of the classic game as the channel cross-elasticity coefficient α and free-riding behavior coefficient λ increase.

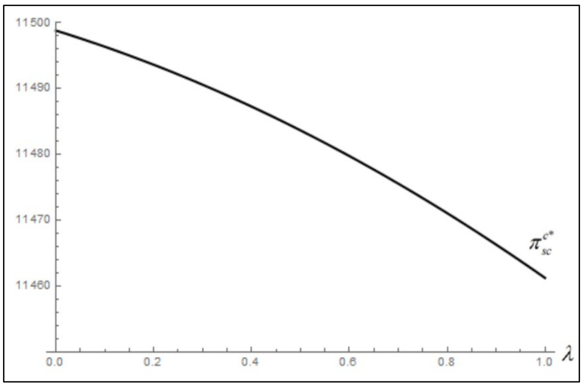

Fig. 9. The free-riding behavior harms the overall profit of the supply chain.

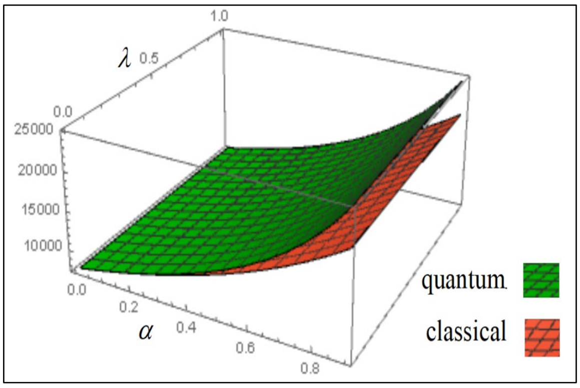

Fig. 10. 3D comparison between the classical game and quantum game.

5. Conclusions

We explore the influences of free-riding behavior of the online direct sales channel on the dual-channel supply chain and compare the result of the classical and the quantum game. The free-riding behavior in the online channel increases its profit by damaging the profit of the retail channel, so the free-riding behavior seriously harms the overall profit of the supply chain. Then, this causes the collapse of overall supply chain cooperation. From the perspective of quantum games, the pricing of the quantum game decision is the same as that of classical game theory when quantum entanglement $\gamma = 0$ under decentralized decision-making. When quantum entanglement γ changes, quantum game pricing also changes accordingly. When the quantum entanglement $\gamma > 0$, the optimal price and profit of quantum games are higher than those of classical games. When $\gamma \to \infty$, the optimal price of the quantum game finally remains in convergence as the degree of quantum entanglement between the retailer and the manufacturer increases. Therefore, the quantum game theory provides more diverse and optimized decision-making methods than the classical game theory.

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Appendix 1. The Proof of Theorem 1.

Proof. Substituting Equations (19) and (20) into Equations (3) and (4) obtains the profit functions of retailer and manufacturer. According to the first derivative condition of the profit functions $\frac{\partial \pi_j}{\partial x_j} = 0$, $j = 0, 1$, the followings are proposed.

$$x_0(-2b_0 \cosh^2 \gamma + 2\alpha \sinh \gamma \cosh \gamma) + x_1(-2b_0 \sinh \gamma \cosh \gamma + \alpha(\cosh^2 \gamma + \sinh^2 \gamma))$$
$$= -[a_0 + (1-\lambda)s] \cosh \gamma + \omega\,(-b_0 \cosh \gamma + \alpha \sinh \gamma), \quad \text{(A1)}$$
$$x_0(-2b_1 \sinh \gamma \cosh \gamma + \alpha(\cosh^2 \gamma + \sinh^2 \gamma)) + x_1(-2b_1 \cosh^2 \gamma + 2\alpha \sinh \gamma \cosh \gamma)$$
$$= -(a_1 + \lambda s) \cosh \gamma + (b_0(\omega - c) + c\alpha) \sinh \gamma - (b_1 c + \alpha(\omega - c)) \cosh \gamma. \quad \text{(A2)}$$

The second derivative condition of the profit functions are

$$\frac{\partial^2 \pi_j}{\partial x_j^2} = (-2b_j \cosh^2 \gamma + 2\alpha \sinh \gamma \cosh \gamma) = -\frac{(b_j-\alpha)e^{2\gamma} + (b_j+\alpha)e^{-2\gamma} + 2}{2} < 0, \quad \text{(A3)}$$

where $j = 0, 1$. Thus, this quantum game model suggests the optimal prices.

The first derivative condition can be written by $BX = \beta$, i.e., $\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$, where

$$B_{11} = 2b_0 \cosh^2 \gamma - 2\alpha \sinh \gamma \cosh \gamma,$$
$$B_{12} = 2b_0 \sinh \gamma \cosh \gamma - \alpha(\cosh^2 \gamma + \sinh^2 \gamma),$$
$$B_{21} = 2b_1 \sinh \gamma \cosh \gamma - \alpha(\cosh^2 \gamma + \sinh^2 \gamma),$$
$$B_{22} = 2b_1 \cosh^2 \gamma - 2\alpha \sinh \gamma \cosh \gamma,$$
$$\beta_1 = [a_0 + (1-\lambda)s] \cosh \gamma - \omega\,(-b_0 \cosh \gamma + \alpha \sinh \gamma),$$
$$\beta_2 = (a_1 + \lambda s) \cosh \gamma - (b_0(\omega - c) + c\alpha) \sinh \gamma + (b_1 c + \alpha(\omega - c)) \cosh \gamma.$$

Since $\cosh^2 \gamma - \sinh^2 \gamma = 1$ and $b_0, b_1 > 1 > \alpha$,

$$|B| = 4b_0 b_1 \cosh^2 \gamma\,(\cosh^2 \gamma - \sinh^2 \gamma) - 2\alpha(b_0 + b_1) \sinh \gamma \cosh \gamma\,(\cosh^2 \gamma - \sinh^2 \gamma)$$
$$-\alpha^2(\cosh^2 \gamma - \sinh^2 \gamma)^2$$
$$= 4b_0 b_1 \cosh^2 \gamma - 2\alpha(b_0 + b_1) \sinh \gamma \cosh \gamma - \alpha^2$$
$$= \left(b_0 b_1 - \frac{\alpha(b_0 + b_1)}{2}\right) e^{2\gamma} + \left(b_0 b_1 + \frac{\alpha(b_0 + b_1)}{2}\right) e^{-2\gamma} + (2b_0 b_1 - \alpha^2) > 0. \quad \text{(A4)}$$

Applying Cramer's Rule, we can obtain the optimal strategies

$$x_0^* = \frac{1}{|B|}\{[2b_1(a_0 + (1-\lambda)s + b_0\omega) + (a_1 + \lambda s + b_1 c)\alpha + (\omega - c)\alpha^2]\cosh^3\gamma$$
$$-[2b_0(a_1 + \lambda s + b_1 c) + (2a_0 + 2(1-\lambda)s + 2b_1\omega + 5b_0\omega - 3b_0 c)\alpha + c\alpha^2]\cosh^2\gamma\sinh\gamma$$
$$+[2b_0^2(\omega - c) + (a_1 + \lambda s + 2b_0 c + b_1 c)\alpha + (3\omega - c)\alpha^2]\cosh\gamma\sinh^2\gamma$$
$$-[b_0(\omega - c)\alpha + c\alpha^2]\sinh^3\gamma\}, \tag{A5}$$

$$x_1^* = \frac{1}{|B|}\{[2b_0(a_1 + \lambda s + b_1 c) + (a_0 + (1-\lambda)s - 2b_0 c + 3b_0\omega)\alpha]\cosh^3\gamma - \omega\alpha^2\sinh^3\gamma$$
$$-[2b_0^2(\omega - c) + 2b_0 b_1\omega + 2b_1 a_0 + 2b_1(1-\lambda)s$$
$$+2(a_1 + \lambda s + b_1 c + b_0 c)\alpha + (3\omega - 2c)\alpha^2)]\cosh^2\gamma\sinh\gamma$$
$$+[(a_0 + (1-\lambda)s + 3b_0\omega - 2b_0 c + 2b_1\omega)\alpha + 2c\alpha^2]\cosh\gamma\sinh^2\gamma\}. \tag{A6}$$

Therefore, the optimal strategic solutions x_0^* and x_1^* of a quantum game enable each game player to have the greatest profit. Substituting the optimal strategic solutions x_0^* and x_1^*, into Equations (17) and (18), the optimal prices of each game player in the quantum game is obtained as follows.

$$p_0^* = \frac{1}{|B|}\{[2b_1(a_0 + (1-\lambda)s + b_0\omega) + (a_1 + \lambda s + b_1 c)\alpha + (\omega - c)\alpha^2]\cosh^2\gamma\left(\cosh^2\gamma - \sinh^2\gamma\right)$$
$$+[(b_0 c - 2b_0\omega - 2b_1\omega - a_0 - (1-\lambda)s)\alpha - c\alpha^2]\cosh\gamma\sinh\gamma\left(\cosh^2\gamma - \sinh^2\gamma\right)$$
$$+\omega\alpha^2\sinh^2\gamma\left(\cosh^2\gamma - \sinh^2\gamma\right)\}, \tag{A7}$$

$$p_1^* = \frac{1}{|B|}\{[2b_0(a_1 + \lambda s + b_1 c) + (a_0 + (1-\lambda)s - 2b_0 c + 3b_0\omega)\alpha]\cosh^2\gamma\left(\cosh^2\gamma - \sinh^2\gamma\right)$$
$$-[2b_0^2(\omega - c) + (a_1 + \lambda s + 2b_0 c + b_1 c)\alpha + (2\omega - c)\alpha^2)]\cosh\gamma\sinh\gamma\left(\cosh^2\gamma - \sinh^2\gamma\right)$$
$$+[b_0(\omega - c)\alpha + c\alpha^2]\sinh^2\gamma\left(\cosh^2\gamma - \sinh^2\gamma\right)\}. \tag{A8}$$

Using $\cosh^2\gamma - \sinh^2\gamma = 1,$ the optimal prices can be obtained as (21) and (22).

$$p_0^* = \frac{1}{|B|}\{[2b_1(a_0 + (1-\lambda)s + b_0\omega) + (a_1 + \lambda s + b_1 c)\alpha + (\omega - c)\alpha^2]\cosh^2\gamma$$
$$+[(b_0 c - 2b_0\omega - 2b_1\omega - a_0 - (1-\lambda)s)\alpha - c\alpha^2]\cosh\gamma\sinh\gamma + \omega\alpha^2\sinh^2\gamma\}, \tag{A9}$$

$$p_1^* = \frac{1}{|B|}\{[2b_0(a_1 + \lambda s + b_1 c) + (a_0 + (1-\lambda)s - 2b_0 c + 3b_0\omega)\alpha]\cosh^2\gamma$$
$$-[2b_0^2(\omega - c) + (a_1 + \lambda s + 2b_0 c + b_1 c)\alpha + (2\omega - c)\alpha^2)]\cosh\gamma\sinh\gamma$$
$$+[\boldsymbol{b_0(\omega - c)\alpha + c\alpha^2}]\,\boldsymbol{\sinh^2\gamma}\}. \tag{A10}$$

Appendix 2. The Proof of Theorem 1.

Proof. Substituting Equations (17), (18), (19) and (20) of the relationship between quantum strategies and prices into equation (5), we can get the overall supply chain profit function. According to the first derivative condition of the profit function $\frac{\partial \pi_{sc}}{\partial x_j}=0$, $j=0,1$, we have

$$x_0(-2b_0\cosh^2\gamma+4\alpha\sinh\gamma\cosh\gamma-2b_1\sinh^2\gamma)$$
$$+x_1(2\alpha\cosh^2\gamma-2(b_0+b_1)\sinh\gamma\cosh\gamma+2\alpha\sinh^2\gamma)$$
$$=-(a_0+(1-\lambda)s+c(b_0-\alpha))\cosh\gamma-(a_1+\lambda s+c(b_1-\alpha))\sinh\gamma, \quad \text{(A11)}$$
$$x_0(2\alpha\cosh^2\gamma-2(b_0+b_1)\sinh\gamma\cosh\gamma+2\alpha\sinh^2\gamma)$$
$$+x_1(-2b_1\cosh^2\gamma+4\alpha\sinh\gamma\cosh\gamma-2b_0\sinh^2\gamma)$$
$$=-(a_0+(1-\lambda)s+c(b_0-\alpha))\sinh\gamma-(a_1+\lambda s+c(b_1-\alpha))\cosh\gamma. \quad \text{(A12)}$$

The first derivative condition can be written by $\hat{B}X=\hat{\beta}$, i.e., $\begin{bmatrix}\hat{B}_{11} & \hat{B}_{12}\\ \hat{B}_{21} & \hat{B}_{22}\end{bmatrix}\begin{bmatrix}x_0\\ x_1\end{bmatrix}=\begin{bmatrix}\hat{\beta}_1\\ \hat{\beta}_2\end{bmatrix}$, where

$$\hat{B}_{11}=2b_0\cosh^2\gamma-4\alpha\sinh\gamma\cosh\gamma+2b_1\sinh^2\gamma,$$
$$\hat{B}_{12}=-2\alpha\cosh^2\gamma+2(b_0+b_1)\sinh\gamma\cosh\gamma-2\alpha\sinh^2\gamma,$$
$$\hat{B}_{21}=-2\alpha\cosh^2\gamma+2(b_0+b_1)\sinh\gamma\cosh\gamma-2\alpha\sinh^2\gamma,$$
$$\hat{B}_{22}=2b_1\cosh^2\gamma-4\alpha\sinh\gamma\cosh\gamma+2b_0\sinh^2\gamma,$$
$$\hat{\beta}_1=(a_0+(1-\lambda)s+c(b_0-\alpha))\cosh\gamma+(a_1+\lambda s+c(b_1-\alpha))\sinh\gamma,$$
$$\hat{\beta}_2=(a_0+(1-\lambda)s+c(b_0-\alpha))\sinh\gamma+(a_1+\lambda s+c(b_1-\alpha))\cosh\gamma.$$

Since $\cosh^2\gamma-\sinh^2\gamma=1$, we have

$$|\,\hat{B}\,|=(4b_0b_1-4\alpha^2)(\cosh^2\gamma-\sinh^2\gamma)=4b_0b_1-4\alpha^2>0. \quad \text{(A13)}$$

From the second derivative condition, we have the Hessian matrix $H=\begin{bmatrix}H_{11} & H_{12}\\ H_{21} & H_{22}\end{bmatrix}$, where

$$H_{11}=-2b_0\cosh^2\gamma+4\alpha\sinh\gamma\cosh\gamma-2b_1\sinh^2\gamma,$$
$$H_{12}=2\alpha\cosh^2\gamma-2(b_0+b_1)\sinh\gamma\cosh\gamma+2\alpha\sinh^2\gamma,$$
$$H_{21}=2\alpha\cosh^2\gamma-2(b_0+b_1)\sinh\gamma\cosh\gamma+2\alpha\sinh^2\gamma,$$
$$H_{22}=-2b_1\cosh^2\gamma+4\alpha\sinh\gamma\cosh\gamma-2b_0\sinh^2\gamma.$$

When $b=\min\{\,b_0,b_1\,\}$, since $b_0,b_1>1>\alpha$,

$$H_{11}\leq-2b\left(\cosh^2\gamma+\sinh^2\gamma\right)+4\alpha\sinh\gamma\cosh\gamma=(-b+\alpha)e^{2\gamma}-(b+\alpha)e^{-2\gamma}<0 \quad \text{(A14)}$$
$$|H|=4(b_0b_1-\alpha^2)(\cosh^2\gamma-\sinh^2\gamma)>0. \quad \text{(A15)}$$

The Hessian matrix is negatively definite, so the overall supply chain profit function has the maximum value. With Cramer's Rule, we obtain the optimal strategies as follows.

$$x_0^{c*} = \frac{1}{|\hat{B}|}\{[2b_1(a_0 + (1-\lambda)s + b_0c) + 2(a_1 + \lambda s)\alpha - 2c\alpha^2]\cosh\gamma\,(\cosh^2\gamma - \sinh^2\gamma)$$

$$+[-2b_0(a_1 + \lambda s + b_1c) - 2(a_0 + (1-\lambda)s)\alpha + 2c\alpha^2)]\sinh\gamma\,(\cosh^2\gamma - \sinh^2\gamma)\}$$

$$= \frac{1}{|\hat{B}|}\{[2b_1(a_0 + (1-\lambda)s + b_0c) + 2(a_1 + \lambda s)\alpha - 2c\alpha^2]\cosh\gamma$$

$$+[-2b_0(a_1 + \lambda s + b_1c) - 2(a_0 + (1-\lambda)s)\alpha + 2c\alpha^2]\sinh\gamma\}, \quad \text{(A16)}$$

$$x_1^{c*} = \frac{1}{|\hat{B}|}\{\,[2b_0(a_1 + \lambda s + b_1c) + 2(a_0 + (1-\lambda)s)\alpha - 2c\alpha^2]\cosh\gamma\,(\cosh^2\gamma - \sinh^2\gamma)$$

$$+[-2b_1(a_0 + (1-\lambda)s + b_0c) - 2(a_1 + \lambda s)\alpha + 2c\alpha^2]\sinh\gamma\,(\cosh^2\gamma - \sinh^2\gamma)\,\}$$

$$= \frac{1}{|\hat{B}|}\{[2b_0(a_1 + \lambda s + b_1c) + 2(a_0 + (1-\lambda)s)\alpha - 2c\alpha^2]\cosh\gamma$$

$$+[-2b_1(a_0 + (1-\lambda)s + b_0c) - 2(a_1 + \lambda s)\alpha + 2c\alpha^2]\sinh\gamma\}. \quad \text{(A17)}$$

Substituting Equations (A7) and (A8) into equations (17) and (18) leads to the follows.

$$p_0^{c*} = \frac{1}{|B|}\{\,[2b_1(a_0 + (1-\lambda)s + b_0c) + 2(a_1 + \lambda s)\alpha - 2c\alpha^2]\left(\cosh^2\gamma - \sinh^2\gamma\right)$$

$$= \frac{b_1(a_0+(1-\lambda)s+b_0c)+\alpha(a_1+\lambda s-c\alpha)}{2(b_0b_1-\alpha^2)} = \bar{p}_0^{c*} \quad \text{(A18)}$$

$$p_1^{c*} = \frac{1}{|B|}\{\,[2b_0(a_1 + \lambda s + b_1c) + 2(a_0 + (1-\lambda)s)\alpha - 2c\alpha^2]\left(\cosh^2\gamma - \sinh^2\gamma\right)$$

$$= \frac{b_0(a_1+\lambda s+b_1c)+\alpha(a_0+(1-\lambda)s-c\alpha)}{2(b_0b_1-\alpha^2)} = \bar{p}_1^{c*}. \quad \text{(A19)}$$

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